

ECE 302 Homework Assignment 1

1. Consider the experiment in which one tosses a fair die and counts the dots on the side facing up.
 - (a) What is the sample space of this experiment? (Your answer should list the elements (outcomes) in the sample space.)
 - (b) What is the event A corresponding to “an even number of dots were counted”? (Your answer should list the elements of the set A , *i.e.* the outcomes contained in the event A .)
 - (c) List the outcomes contained in the event A^c . Also characterize the event A^c in words.
2. A fair die is tossed twice and the numbers n_i of dots facing up are noted, ($i \in \{1, 2\}$).
 - (a) What is the sample space?
 - (b) What are the elements of the event A corresponding to “total number of dots is odd”?
 - (c) What are the elements of the event B corresponding to “both tosses are odd.”
 - (d) What are the elements of the event $A \cap B^c$. Also describe this event in words.
 - (e) Let C correspond to the event “numbers of dots observed in the two tosses differ by 1”. (Be sure to state precisely how you have defined C .) Find $A \cap C$.
3. Consider the experiment in which one tosses two dice and records the total number of dots facing up.
 - (a) What is the sample space?
 - (b) What are the elements of the event A corresponding to “total number of dots counted is even”?
 - (c) Express each of the outcomes in this sample space in terms of the elements of the sample space of the previous problem. (Think of the two throws of the one die in the previous problem as being throws of two separate dice.)
4. Consider tossing a die and recording the number N_1 of dots facing up, then choosing an integer N_2 between 1 and N_1 at random (meaning that each integer is equally likely to be chosen).
 - (a) Find the sample space.
 - (b) Find the set of outcomes in the event “die shows four dots facing up.”
 - (c) Find the set of outcomes in the event “ $N_2 = 3$ ”.

- (d) Find the set of outcomes in the event " $N_2 = 6$ ".
5. A desk drawer contains five pens, three of which are dry.
- (a) Suppose that one selects and tests pens at random, one by one, until a good pen is found and notes the sequence of test results. What is the sample space?
 - (b) Suppose that only the number of pens tested, as opposed to the sequence of test results, is noted. What is the sample space?
 - (c) Suppose that the pens are selected one by one and tested until both good pens have been identified, and the sequence of test results is noted. What is the sample space?
 - (d) Suppose that the pens are selected one by one and tested until both good pens have been identified, and the number of pens tested is noted. What is the sample space?
6. A student wakes up at time T_1 and goes to sleep at some later time T_2 . Assume that $T_2 - T_1 \leq 24$.
- (a) What is the sample space? Sketch it in the x - y plane. (Plot T_1 on the x -axis and T_2 on the y -axis.)
 - (b) What are the outcomes contained in the event A "student is still awake 14 hours after waking up"? Sketch the region on the plane corresponding to this event.
 - (c) What are the outcomes in the event B "student is asleep more than awake"? Sketch the region in the plane corresponding to this event.
 - (d) Sketch the region corresponding to $A^c \cap B$ and describe the corresponding event in words.
7. A die is tossed and the number of dots facing up is noted.
- (a) Assume the die is fair. Find the probability of each possible outcome in the sample space occurring.
 - (b) Now assume instead that the face with a single dot is twice as likely to be facing up as the rest of the faces. (Assume that the others are equally likely.) Find the probabilities as before.
 - (c) Now assume instead that the face with a single dot is twice as likely to be facing up as any of the other faces. (Assume that the others are equally likely.) Find the probabilities as before.
 - (d) Find the probabilities that the outcome of a toss is even under the three different assumptions above.

8. A die is tossed twice and the number of dots facing up noted in the order of the tosses. Assuming that all outcomes are equally likely to occur, find the probabilities of the following events:
 - (a) A_k : “the sum of the two values noted is k ”, for each $k \in \{2, 3, \dots, 12\}$.
 - (b) B : “the two values noted are different.”
9. An experiment has sample space $S = \{a, b, c\}$. Given $P[\{a, c\}] = 5/8$ and $P[\{b, c\}] = 7/8$, use the axioms of probability to find the probability of each outcome in S occurring.
10. Explain why the probability that exactly one of events A or B (defined on the same sample space) occurs is $P[A] + P[B] - 2P[A \cap B]$.
11. Consider selecting two numbers at random from the interval $[0, 1]$. Find the probability that they differ by more than $1/2$.